MATH 147 QUIZ 2 SOLUTIONS

1. For the function $f(x, y)$, the point $(a, b) \in \mathbb{R}^2$ in the domain of $f(x, y)$, define what it means for $f(x, y)$ to be differentiable at (a, b). (2 Points)

   We say $f(x, y)$ is *differentiable* at (a, b) if $f_x(a, b)$ and $f_y(a, b)$ exist and for $L(x, y) := f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b)$,

   $$\lim_{(x,y)\to(a,b)} \frac{f(x, y) - L(x, y)}{\|(x, y) - (a, b)\|} = 0.$$

2. Find the tangent plane to the graph of $f(x, y) = \ln\big(10x^2 + 2y^2 + 1\big)$ at the point $(0, 0, f(0, 0))$. You may assume the tangent plane exists at $(0, 0, f(0, 0))$. (4 points)

   Recall that the equation of a tangent plane is given by $L(x, y) = f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b)$. Thus, we begin by finding the partial derivatives. We have

   $$f_x(x, y) = \frac{1}{10x^2 + 2y^2 + 1} \cdot 20x, \quad \text{and} \quad f_y(x, y) = \frac{1}{10x^2 + 2y^2 + 1} \cdot 4y.$$

   Thus, we have $f_x(0, 0) = f_y(0, 0) = 0$. In addition, we have that $f(0, 0) = \ln(1) = 0$. Thus, the tangent plane is given by

   $$L(x, y) = 0(x) + 0(y) + 0 = 0.$$

   That is, the tangent plane to this surface at the origin is the flat plane defined by $z = 0$.

3. For $f(x, y) = \begin{cases} \frac{3x^2y - y^3}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0), \end{cases}$ find a formula for $f_x(x, y)$. (4 points)

   As this is a rational function in x, it is differentiable everywhere except possibly for when $x^2 + y^2 = 0$. Thus, we first see what the function is doing away from the origin. Use the quotient rule to see

   $$f_x(x, y) = \frac{(x^2 + y^2)(6xy) - (3x^2y - y^3)(2x)}{(x^2 + y^2)^2} = \frac{8xy^3}{(x^2 + y^2)^2}.$$

   Next, we use the limit definition of derivative to see what is happening at the origin. We should have

   $$f_x(0, 0) = \lim_{h\to 0} \frac{f(h, 0) - f(0, 0)}{h} = \lim_{h\to 0} \frac{\frac{0}{h^2} - 0}{h} = \lim_{h\to 0} \frac{0}{h} = \lim_{h\to 0} 0 = 0.$$

   Thus, we can say that

   $$f_x(x, y) = \begin{cases} \frac{8xy^3}{(x^2 + y^2)^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$